

# Nonequilibrium Phase Transition in the Kinetic Ising model: Absence of tricritical behaviour in presence of impurities

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The nonequilibrium dynamic phase transition, in the two dimensional *site diluted* kinetic Ising model in presence of an oscillating magnetic field, has been studied by Monte Carlo simulation. The projections of dynamical phase boundary *surface* are drawn in the planes formed by the dilution and field amplitude and the plane formed by temperature and field amplitude. The tricritical behaviour is found to be *absent* in this case which was observed in the pure system.

## I. Introduction.

Though the Ising model was proposed nearly three quarters of a century ago its dynamical aspects are still under active investigation [1]. Nowadays, the study of the dynamics of Ising models in presence of time varying magnetic field, became an active and interesting area of modern research. The dynamical response of the Ising system in presence of an oscillating magnetic field has been studied extensively by computer simulation [2, 3, 4, 5, 8, 9] in the last few years. The dynamical hysteretic response [2, 3, 4] and the nonequilibrium dynamical phase transition [5, 8, 9] are two important aspects of the dynamic response of the kinetic Ising model in presence of an oscillating magnetic field.

Tome and Oliviera [5] first studied the dynamic transition in the kinetic Ising model in presence of a sinusoidally oscillating magnetic field. They solved the mean field (MF) dynamic equation of motion (for the average

magnetisation) of the kinetic Ising model in presence of a sinusoidally oscillating magnetic field. By defining the order parameter as the time averaged magnetisation over a full cycle of the oscillating magnetic field they showed that the order parameter vanishes depending upon the value of the temperature and the amplitude of the oscillating field. Precisely, in the field amplitude and temperature plane they have drawn a phase boundary separating dynamic ordered (nonzero value of order parameter) and disordered (order parameter vanishes) phase. They [5] have also observed and located a *tricritical point* (TCP), (separating the nature (discontinuous/continuous) of the transition) on the phase boundary line. It was confirmed later by Monte Carlo study [6] and by solving meanfield differential equation [7] of kinetic Ising model.

Since, this transition exists even in the static (zero frequency) limit such a transition, observed [5] from the solution of mean field dynamical equation, is not dynamic in true sense. This is because, for the field amplitude less than the coercive field (at temperature less than the transition temperature without any field), the response magnetisation varies periodically but asymmetrically even in the zero frequency limit; the system remains locked to one well of the free energy and cannot go to the other one, in the absence of noise or fluctuation.

Lo and Pelcovits [8] first attempted to study the dynamic nature of this phase transition (incorporating the effect of fluctuation) in the kinetic Ising model by Monte Carlo (MC) simulation. In this case, the transition disappears in the zero frequency limit; due to the presence of fluctuations, the magnetisation flips to the direction of the magnetic field and the dynamic order parameter (time averaged magnetisation) vanishes. However, they [8] have not reported any precise phase boundary. Acharyya and Chakrabarti [9] studied the nonequilibrium dynamic phase transition in the kinetic Ising model in presence of oscillating magnetic field by extensive MC simulation. It was also noticed by them [9] that this dynamic phase transition is associated with the breaking of the symmetry of the dynamic hysteresis ( $m - h$ ) loop. In the dynamically disordered (value of order parameter vanishes) phase the corresponding hysteresis loop is symmetric, and loses its symmetry in the ordered phase (giving nonzero value of dynamic order parameter). They have [9] also studied the temperature variation of the ac susceptibility components near the dynamic transition point. They observed that the imaginary (real) part of the ac susceptibility gives a peak (dip) near the dynamic transition point (where the dynamic order parameter vanishes). The conclusions were: (i) this is a distinct signal of phase transition and (ii) this is an indication of the thermodynamic nature of the phase transition.

It may be mentioned here that the statistical distribution of dynamic order parameter has been studied by Sides et al [10]. The nature of the distribution changes near the dynamic transition point. They have also observed

[10] that the fluctuation of the hysteresis loop area becomes considerably large near the dynamic transition point.

Very recently, the relaxation behaviour, of the dynamic order parameter, near the transition point, has been studied [11] by MC simulation and solving[12] meanfield dynamic equation. It has been observed that the relaxation is Debye type and the relaxation time diverges near the transition point. The 'specific heat' and the 'susceptibility' also diverge [13] near the transition point in a similar manner with that of fluctuations of order parameter and fluctuation of energy respectively.

The tricritical point was observed in the case of pure system[6]. In this paper, the dynamic phase transition has been studied in the site *diluted* (by nonmagnetic impurities) kinetic Ising model by MC simulation. The phase boundaries are plotted in the planes formed by the field amplitude and the temperature and in the plane formed by the impurity concentration and the field amplitude. The paper is organised as follows: in section II the model and the simulation scheme are discussed, the MC results are given in section III and the paper ends with a summary of the work in section IV.

## II. The Model and the simulation scheme

The Hamiltonian, of a site diluted Ising model (with ferromagnetic nearest neighbour interaction) in presence of a time varying magnetic field, can be written as

$$H = - \sum_{\langle ij \rangle} J_{ij} s_i^z s_j^z c_j - h(t) \sum_i s_i^z c_i. \quad (1)$$

Here,  $s_i^z (= \pm 1)$  is Ising spin variable,  $J_{ij}$  is the interaction strength,  $c_i (= 0$  or  $1)$  represents the site ( $i$ ) which is either occupied ( $c_i = 1$ ) or vacant ( $c_i = 0$ ).  $h(t) = h_0 \cos(\omega t)$  is the oscillating magnetic field, where  $h_0$  and  $\omega$  are the amplitude and the frequency respectively of the oscillating field. The system is in contact with an isothermal heat bath at temperature  $T$ . For simplicity all  $J_{ij}$  and the value of the Boltzmann constant are taken equal to unity. The boundary condition is periodic.

A square lattice of linear size  $L (= 100)$  has been considered. The lattice sites are randomly occupied by magnetic sites with a finite probability  $p$ . So, the degree of dilution or the concentration of (nonmagnetic) impurities, is  $q = 1 - p$ . At any finite temperature  $T$  and for a fixed frequency ( $\omega$ ) and amplitude ( $h_0$ ) of the field, the dynamics of this system has been studied here by Monte Carlo simulation using Metropolis single spin-flip dynamics. Each lattice site is updated here sequentially and one such full scan over the lattice is defined as the time unit (Monte Carlo step per spin or MCS) here. The initial configuration has been chosen such that the all spins are directed upward. The instantaneous magnetisation (per site),  $m(t) = (1/L^2) \sum_i s_i^z c_i$  has been calculated. From the instantaneous magnetisation, the dynamic

order parameter  $Q = \frac{\omega}{2\pi} \oint m(t) dt$  (time averaged magnetisation over a full cycle of the oscillating field) is calculated. This dynamic order parameter is a function of temperature ( $T$ ), field amplitude ( $h_0$ ) and the impurity concentration ( $q$ ), i.e.,  $Q = Q(T, h_0, q)$ . Each value of  $Q$  has been calculated by averaging over 25 number of initial impurity realisations. The frequency of the oscillating magnetic field used here is equal to 0.0628.

### III. Results

It has been observed that  $Q = Q(T, h_0, q)$  is nonzero for a finite set of values of  $h_0, T$  and  $q$ , and  $Q$  vanishes elsewhere. In the space formed by  $h_0, T$  and  $q$ , there is a surface which divides the  $Q = 0$  region from  $Q \neq 0$  region. Fig. 1 shows the schematic diagram of this phase surface.

Previously, a number of numerical studies [4,6,8] are performed regarding the dynamic transition in the  $q = 0$  plane (i.e., projection of this phase surface on  $h_0 - T$  plane). In that case, it was observed that in the  $h_0 - T$  plane there is a distinct phase boundary below which  $Q$  is nonzero and above which  $Q$  vanishes. There is a *tricritical point* on the phase boundary which separates the nature (discontinuous/continuous) of this transition.

Fig. 2a shows the dynamic phase boundary in the  $h_0 - T$  plane for different values of the impurity concentration. It has been observed that as the impurity concentration increases the phase boundary shrinks inward. In this case, the entire phase boundary has been scanned and the transition observed is always *continuous*. Unlike the earlier case [9], no such *tricritical point* is observed here. Fig. 2b shows the temperature variations of the dynamic order parameter (i.e.,  $Q$  versus  $T$ ) for two different values of  $h_0$  in the case of very weak disorder (impurity).

A similar kind of dynamic phase boundary has been obtained in the  $h_0 - q$  plane (i.e., the projection of the phase surface on  $h_0 - q$  plane). Fig 3a. shows the phase boundaries in the  $h_0 - q$  plane for different values of temperatures. Like the earlier case, here also, as the temperature increases the phase boundary shrinks inward. Here also the transition is continuous along the entire phase boundary. Two typical transitions (fall of order parameter with respect to the impurity concentrations) are shown in Fig. 3b.

### VI. Summary

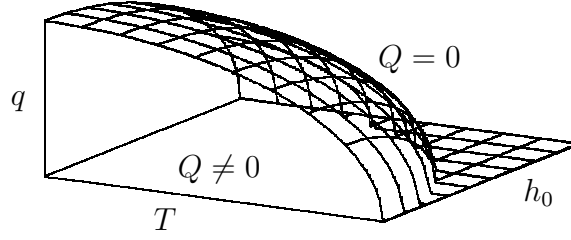
The nonequilibrium dynamic phase transition, in the *site diluted* kinetic Ising model in presence of oscillating magnetic field, is studied by Monte Carlo simulation.

The value of the dynamic order parameter gets nonzero below a boundary *surface* in  $T, h_0$  and  $q$  space, and above the surface it vanishes. The projections of this surface on  $h_0 - T$  plane and on  $h_0 - q$  plane are drawn.

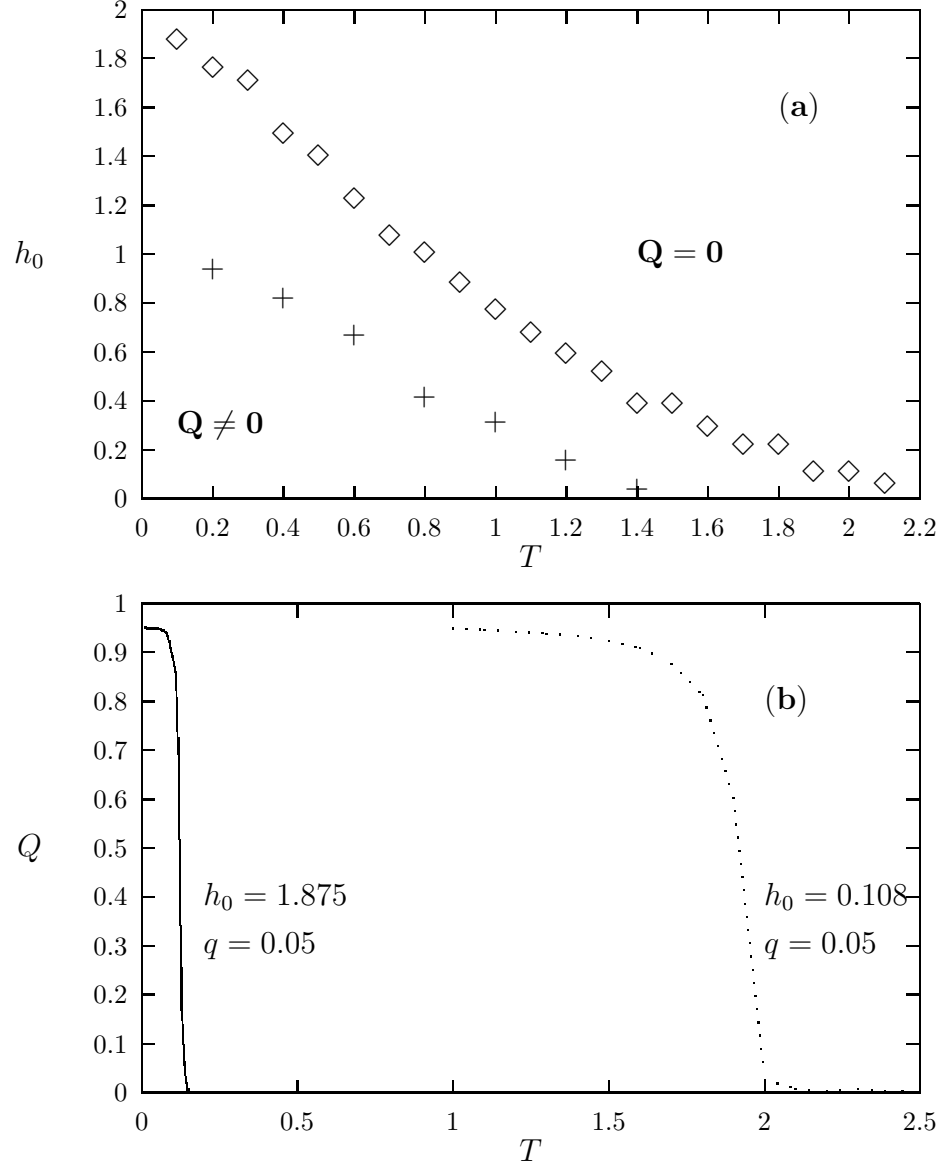
The nature of the transition observed here is continuous along the entire phase boundary in the  $h_0 - T$  plane with very small impurity concentrations. This is unlike the case observed earlier [6, 7] in the pure sample. A similar kind of transition is observed for a fixed temperature with varying impurity concentrations. Here also no tricritical behaviour was observed. The studies reported here are mostly observational, no attempt has been made to understand these phenomena from the knowledge of the theoretical background.

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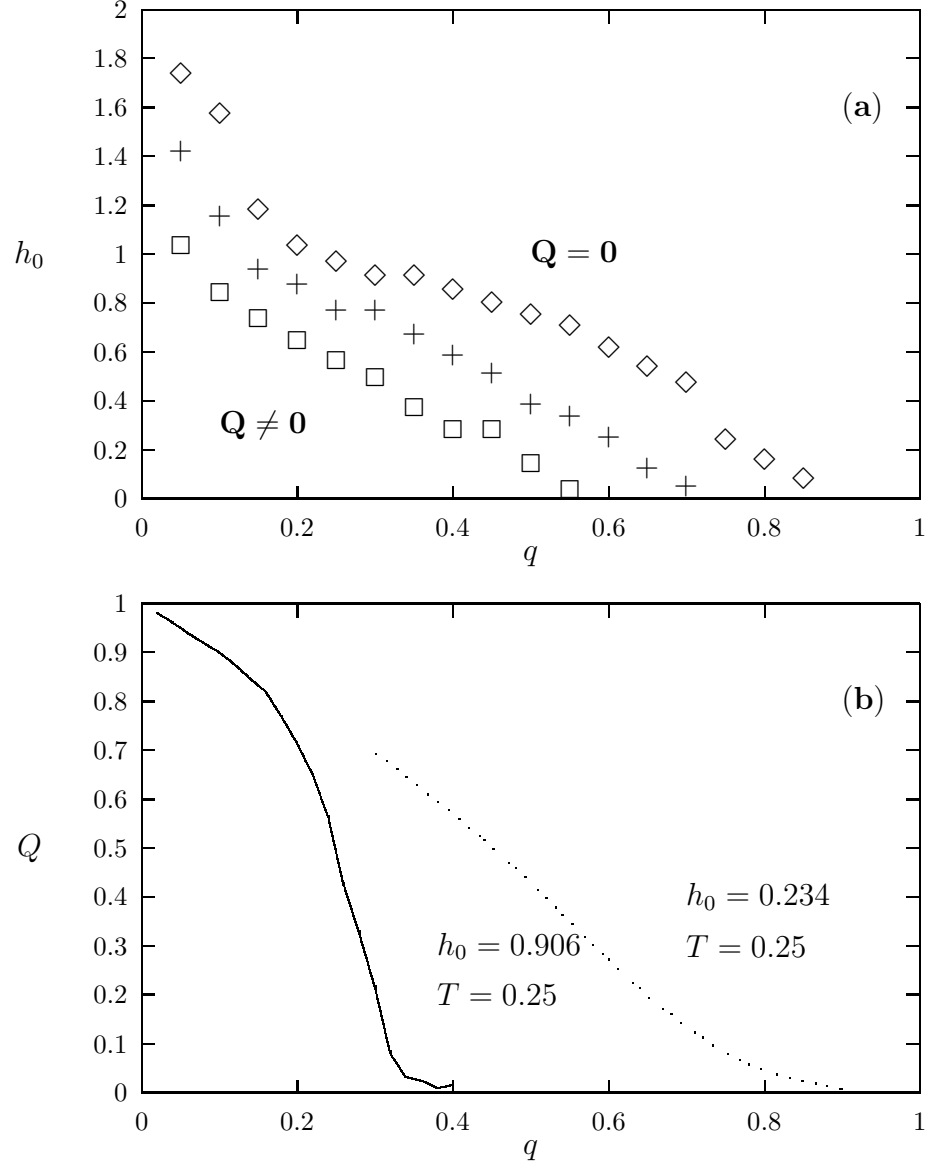
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**Fig. 1.** Schematic diagram of dynamic phase boundary surface in the space formed by  $T, h_0$  and  $q$ . Below this surface  $Q \neq 0$  and above the surface  $Q = 0$ .



**Fig. 2.** (a) Projections of dynamic phase surface on the  $h_0 - T$  plane. ( $\diamond$ ) represents  $q = 0.05$  and (+) represents  $q = 0.3$ . (b) Temperature variations of dynamic order parameter ( $Q$ ) for two different values of field amplitudes ( $h_0$ ).



**Fig. 3.** (a) Projections of dynamic phase surface on the  $h_0 - q$  plane. ( $\diamond$ ) represents  $T = 0.25$ , ( $+$ ) represents  $T = 0.50$  and ( $\square$ ) represents  $T = 0.75$ . (b) Variations of dynamic order parameter ( $Q$ ) with respect to impurity concentration ( $q$ ) for two different values of field amplitudes ( $h_0$ ).